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# Learning to Execute

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## Abstract

Recurrent Neural Networks (RNNs) with Long-Short Term Memory units (LSTM) are widely used because they are expressive and are easy to train. Our interest lies in empirically evaluating the expressiveness and the learnability of LSTMs by training them to evaluate short computer programs, a problem that has traditionally been viewed as too complex for neural networks. We consider a simple class of programs that can be evaluated with a single left-to-right pass using constant memory. Our main result is that LSTMs can learn to map the character-level representations of such programs to their correct outputs. Notably, it was necessary to use curriculum learning, and while conventional curriculum learning proved ineffective, we developed a new variant of curriculum learning that improved our networks' performance in all experimental conditions.

## 1. Introduction

Execution of computer programs requires dealing with multiple nontrivial concepts. To execute a program, a system has to understand numerical operations, the branching of if-statements, the assignments of variables, the compositionality of operations, and many more.

We show that Recurrent Neural Networks (RNN) with Long-Short Term Memory (LSTM) units can accurately evaluate short simple programs. The LSTM reads the program character-by-character and computes the program's output. We considered a constrained set of computer programs that can be evaluated in linear time and constant memory because the LSTM reads the program only once and its memory is small (Section 3). Indeed, the runtime of

the LSTM is linear in the size of the program, so it cannot simulate programs that have a greater minimal runtime.

It is difficult to train LSTMs to execute computer programs, so we used curriculum learning to simplify the learning problem. We design a curriculum procedure which outperforms both conventional training that uses no curriculum learning (*baseline*) as well as *naive* curriculum learning (Bengio et al., 2009) (Section 4). We provide a plausible explanation for the effectiveness of our procedure relative to *naive* curriculum learning (Section 7).

Finally, in addition to curriculum learning strategies, we examine two simple input transformations that further simplify the learning problem. We show that, in many cases, reversing the input sequence (Sutskever et al., 2014) and replicating the input sequence improves the LSTM's performance on a memorization task (Section 3.1).

## 2. Related work

There has been related research that used Tree Neural Network (sometimes known as Recursive Neural Networks) to evaluate symbolic mathematical expressions and logical formulas (Zaremba et al., 2014a; Bowman et al., 2014; Bowman, 2013), which is close in spirit to our work. However, Tree Neural Networks require parse trees, and in aforementioned work they process operations on the level of words, so each operation is encoded with its index. Computer programs are also more complex than mathematical or logical expressions due to branching, looping, and variable assignment.

From a methodological perspective, we formulate the program evaluation task as a language modeling problem on a sequence (Mikolov, 2012; Sutskever, 2013; Pascanu et al., 2013). Other interesting applications of recurrent neural networks includes speech recognition (Robinson et al., 1996; Graves et al., 2013), machine translation (Cho et al., 2014; Sutskever et al., 2014), handwriting recognition (Pham et al., 2013; Zaremba et al., 2014b), and many more.

(Maddison & Tarlow, 2014) learned a language model on parse trees, and (Mou et al., 2014) predicted whether two programs are equivalent or not. Both of these approaches require parse trees, while we learn from a program character level sequence.

Predicting program output requires that the model deals with long term dependencies that arise from variable assignment. Thus we chose to use Recurrent Neural Networks with Long Short Term Memory units (Hochreiter & Schmidhuber, 1997), although there are many other RNN variants that perform well on tasks with long term dependencies (Cho et al., 2014; Jaeger et al., 2007; Koutník et al., 2014; Martens, 2010; Bengio et al., 2013).

Initially, we found it difficult to train LSTMs to accurately evaluate programs. The compositional nature of computer programs suggests that the LSTM would learn faster if we first taught it the individual operators separately and then taught the LSTM how to combine them. This approach can be implemented with curriculum learning (Bengio et al., 2009; Kumar et al., 2010; Lee & Grauman, 2011), which prescribes gradually increasing the “difficulty level” of the examples presented to the LSTM, and is partially motivated by fact that humans and animals learn much faster when their instruction provides them with hard but manageable exercises. Unfortunately, we found the *naive* curriculum learning strategy of Bengio et al. (2009) to be generally ineffective and occasionally harmful. One of our key contributions is the formulation of a new curriculum learning strategy that substantially improves the speed and the quality of training in every experimental setting that we considered.

### 3. Subclass of programs

We train RNNs on class of simple programs that can be evaluated in  $O(n)$  time and constant memory. This restriction is dictated by the computational structure of the RNN itself, at it can only do a single pass over the program using a very limited memory. Our programs use the Python syntax and are based on a small number of operations and their composition (*nesting*). We consider the following operations: addition, subtraction, multiplication, variable assignment, if-statement, and for-loops, although we forbid double loops. Every program ends with a single “print” statement that outputs a number. Several example programs are shown in Figure 1.

We select our programs from a family of distributions parameterized by *length* and *nesting*. The *length* parameter is the number of digits in numbers that appear in the programs (so the numbers are chosen uniformly from  $[1, 10^{\text{length}}]$ ). For example, the programs are generated with *length* = 4 (and *nesting* = 3) in Figure 1.

**Input:**

```
j=8584
for x in range(8) :
    j+=920
    b=(1500+j)
    print((b+7567))
```

**Target:** 25011.

**Input:**

```
i=8827
c=(i-5347)
print((c+8704) if 2641<8500 else
      5308)
```

**Target:** 1218.

Figure 1. Example programs on which we train the LSTM. The output of each program is a single number. A “dot” symbol indicates the end of a number and has to be predicted as well.

We are more restrictive with multiplication and the ranges of for-loop, as these are much more difficult to handle. We constrain one of the operands of multiplication and the range of for-loops to be chosen uniformly from the much smaller range  $[1, 4 \cdot \text{length}]$ . This choice is dictated by the limitations of our architecture. Our models are able to perform linear-time computation while generic integer multiplication requires superlinear time. Similar restrictions apply to for-loops, since nested for-loops can implement integer multiplication.

The *nesting* parameter is the number of times we are allowed to combine the operations with each other. Higher value of *nesting* results in programs with a deeper parse tree. Nesting makes the task much harder for our LSTMs, because they do not have a natural way of dealing with compositionality, in contrast to Tree Neural Networks. It is surprising that they are able to deal with nested expressions at all.

It is important to emphasize that the LSTM reads the input one character at a time and produces the output character by character. The characters are initially meaningless from the model’s perspective; for instance, the model does not know that “+” means addition or that 6 is followed by 7. Indeed, scrambling the input characters (e.g., replacing “a” with “q”, “b” with “w”, etc.,) would have no effect on the model’s ability to solve this problem. We demonstrate the difficulty of the task by presenting an input-output example with scrambled characters in Figure 2.

<b>Input:</b>
vqppkn
sqdvvf1jmnc
y2vxdddsepnimcbvubkomhrpliibtwztbljipcc
<b>Target:</b> hkhpg

Figure 2. An example program with scrambled characters. It helps illustrate the difficulty faced by our neural network.

### 3.1. Memorization Task

In addition to program evaluation, we also investigate the task of memorizing a random sequence of numbers. Given an example input 123456789, the LSTM reads it one character at a time, stores it in memory, and then outputs 123456789 one character at a time. We present and explore two simple performance enhancing techniques: input reversing (from Sutskever et al. (2014)) and input doubling.

The idea of input reversing is to reverse the order of the input (987654321) while keeping the desired output unchanged (123456789). It seems to be a neutral operation as the average distance between each input and its corresponding target did not become shorter. However, input reversing introduces many short term dependencies that make it easier for the LSTM to start making correct predictions. This strategy was first introduced for LSTMs for machine translation by Sutskever et al. (2014).

The second performance enhancing technique is input doubling, where we present the input sequence twice (so the example input becomes 123456789; 123456789), while the output is unchanged (123456789). This method is meaningless from a probabilistic perspective as RNNs approximate the conditional distribution  $p(y|x)$ , yet here we attempt to learn  $p(y|x, x)$ . Still, it gives noticeable performance improvements. By processing the input several times before producing an output, the LSTM is given the opportunity to correct the mistakes it made in the earlier passes.

## 4. Curriculum Learning

Our program generation scheme is parametrized by  $length$  and  $nesting$ . These two parameters allow us control the complexity of the program. When  $length$  and  $nesting$  are large enough, the learning problem nearly intractable. This indicates that in order to learn to evaluate programs of a given  $length = a$  and  $nesting = b$ , it may help to first learn to evaluate programs with  $length \ll a$  and  $nesting \ll b$ . We compare the following curriculum learning strategies:

**No curriculum learning (baseline)** The baseline approach does not use curriculum learning. This means that we

generate all the training samples with  $length = a$  and  $nesting = b$ . This strategy is most “sound” from statistical perspective, as it is generally recommended to make the training distribution identical to test distribution.

### Naive curriculum strategy (*naive*)

We begin with  $length = 1$  and  $nesting = 1$ . Once learning stops making progress, we increase  $length$  by 1. We repeat this process until its  $length$  reaches  $a$ , in which case we increase  $nesting$  by one and reset  $length$  to 1.

We can also choose to first increase  $nesting$  and then  $length$ . However, it does not make a noticeable difference in performance. We skip this option in the rest of paper, and increase  $length$  first in all our experiments. This strategy is has been examined in previous work on curriculum learning (Bengio et al., 2009). However, we show that often it gives even worse performance than *baseline*.

### Mixed strategy (*mix*)

To generate a random sample, we first pick a random  $length$  from  $[1, a]$  and a random  $nesting$  from  $[1, b]$  independently for every sample. The Mixed strategy uses a balanced mixture of easy and difficult examples, so at any time during training, a sizable fraction of the training samples will have the appropriate difficulty for the LSTM.

### Combining the mixed strategy with naive curriculum strategy (*combined*)

This strategy combines the *mix* strategy with the *naive* strategy. In this approach, every training case is obtained either by the *naive* strategy or by the *mix* strategy. As a result, the *combined* strategy always exposes the network at least to some difficult examples, which is the key way in which it differs from the *naive* curriculum strategy. We noticed that it reliably outperformed the other strategies in our experiments. We explain why our new curriculum learning strategies outperform the *naive* curriculum strategy in Section 7.

We evaluate these four strategies on the program evaluation task (Section 6.1) and on the memorization task (Section 6.2).

## 5. RNN with LSTM cells

In this section we briefly describe the deep LSTM (Section 5.1). All vectors are  $n$ -dimensional unless explicitly stated otherwise. Let  $h_t^l \in \mathbb{R}^n$  be a hidden state in layer  $l$  in timestep  $t$ . Let  $T_{n,m} : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a biased linear mapping ( $x \rightarrow Wx + b$  for some  $W$  and  $b$ ). We let  $\odot$  be element-wise multiplication and let  $h_t^0$  be the input at timestep  $k$ . We use the activations at the top layer  $L$  (namely  $h_t^L$ ) to predict  $y_t$  where  $L$  is the depth of our LSTM.

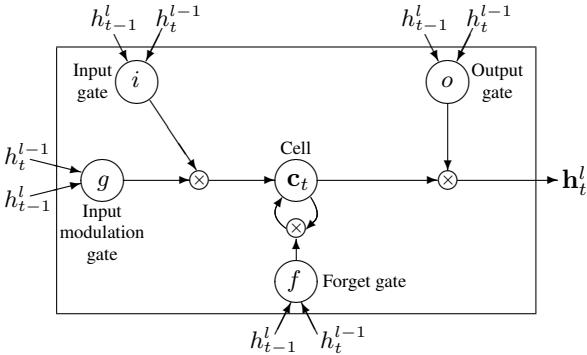


Figure 3. A graphical representation of the LSTM memory cells used in this paper (they differ in minor ways from Graves (2013)).

### 5.1. Long-short term memory units

The structure of the LSTM allows it to learn on problems with long term dependencies relatively easily. The “long term” memory is stored in a vector of *memory cells*  $c_t^l \in \mathbb{R}^n$ . Although many LSTM architectures differ in their connectivity structure and activation functions, all LSTM architectures have memory cells that are suitable for storing information for long periods of time. We used an LSTM described by the following equations (from Graves et al. (2013)):

$$\text{LSTM} : h_t^{l-1}, h_{t-1}^l, c_{t-1}^l \rightarrow h_t^l, c_t^l$$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} T_{2n,4n} \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$

In these equations, the nonlinear functions *sigm* and *tanh* are applied elementwise. Figure 3 shows the LSTM equations (the figure is taken from Zaremba et al. (2014b)).

## 6. Experiments

In this section, we report the results of our curriculum learning strategies on the program evaluation and memorization tasks. In both experiments, we used the same LSTM architecture.

Our LSTM has two layers and is unrolled for 50 steps in both experiments. It has 400 units per layer and its parameters are initialized uniformly in  $[-0.08, 0.08]$ . We initialize the hidden states to zero. We then use the final hidden states of the current minibatch as the initial hidden state of the subsequent minibatch. The size of minibatch is 100. We clip the norm of the gradients (normalized by minibatch

size) at 5 (Mikolov et al., 2010). We keep the learning rate equal to 0.5 until we reach the target *length* and *nesting* (we only vary the *length*, i.e., the number of digits, in the memorization task). After reaching the target accuracy we decrease the learning rate by 0.8. We keep the learning rate on the same level until there is no improvement on the training set. We decrease it again, when there is no improvement on training set. We begin training with *length* = 1 and *nesting* = 1 (or *length*=1 for the memorization task).

To prevent training samples from being repeated in the test set, we enforced that the training, validation, and test sets are disjoint.

### 6.1. Program Evaluation Results

We train our LSTMs using the four strategies described in Section 4:

- No curriculum learning (*baseline*),
- Naive curriculum strategy (*naive*)
- Mixed strategy (*mix*), and
- Combined strategy (*combined*).

Figure 4 shows the absolute performance of the *baseline* strategy (training using target test data distribution), and of the best performing strategy, *combined*. Moreover, Figure 5 shows the performance of all strategies relative to *baseline*. Finally, we provide several example predictions on test data in Figure 6.

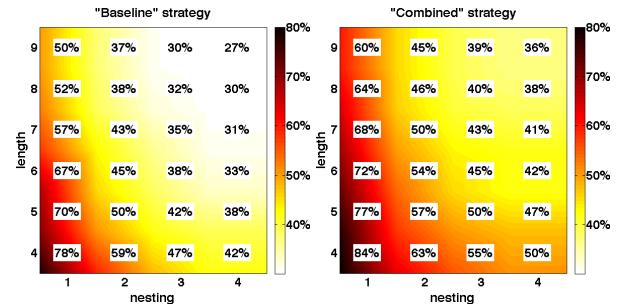


Figure 4. Absolute prediction accuracy of the *baseline* strategy and of the *combined* strategy (see Section 4) on the program evaluation task. Deeper nesting and larger length make the task more difficult. Overall, the *combined* strategy outperformed the *baseline* strategy in every setting.

### 6.2. Memorization Results

Recall that the task is to copy a sequence of input values. Namely, given an input such as 123456789, the goal is to produce the output 123456789. The model accesses one input character at the time and has to produce the output

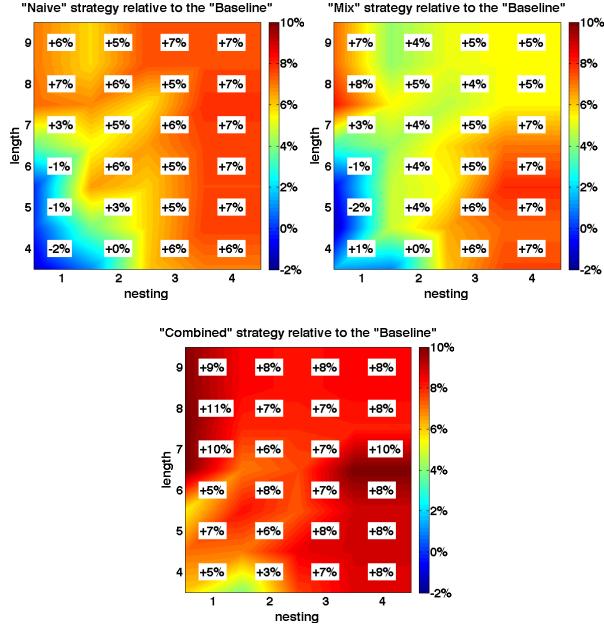


Figure 5. Relative prediction accuracy of the different strategies with respect to the *baseline* strategy. The *Naive* curriculum strategy was found to sometime perform **worse** than *baseline*. A possible explanation is provided in Section 7. The *combined* strategy outperforms all other strategies in every configuration.

only after holding the entire input in its memory. This task gives us insights into the LSTM’s ability to memorize and remember information. We have evaluated our model on sequences of lengths ranging from 5 to 65. We use the four curriculum strategies of Section 4. In addition, we investigate two strategies to modify the input which boost performance:

- inverting input (Sutskever et al., 2014)
- doubling input

Both strategies are described in Section 3.1. Figure 7 shows the absolute performance of the *baseline* strategy and of the *combined* strategy. It is clear that the *combined* strategy outperforms every other strategy. Each graph contains 4 settings, which correspond to the possible combinations of input inversion and input doubling. The result clearly shows that the simultaneously doubling and reversing the input achieves the best results.

## 7. Hidden State Allocation Hypothesis

Our experimental results suggest that a proper curriculum learning strategy is critical for achieving good performance on very hard problems where conventional stochastic gradient descent (SGD) performs poorly. The results on both

**Input:**  
`f=(8794 if 8887<9713 else (3*8334))  
 print ((f+574))`  
**Target:** 9368.  
**Model prediction:** 9368.

**Input:**  
`j=8584  
 for x in range(8):  
 j+=920  
 b=(1500+j)  
 print ((b+7567))`  
**Target:** 25011.  
**Model prediction:** 23011.

**Input:**  
`c=445  
 d=(c-4223)  
 for x in range(1):  
 d+=5272  
 print ((8942 if d<3749 else 2951))`  
**Target:** 8942.  
**Model prediction:** 8942.

**Input:**  
`a=1027  
 for x in range(2):  
 a+=(402 if 6358>8211 else 2158)  
 print (a)`  
**Target:** 5343.  
**Model prediction:** 5293.

Figure 6. Sample predictions generated by our model trained with the *combined* strategy. Here *length* is 4 and *nesting* is 3. The model makes interesting mistakes. For instance, it occasionally makes off-by-one errors yet it has no built-in notion of integer-order.

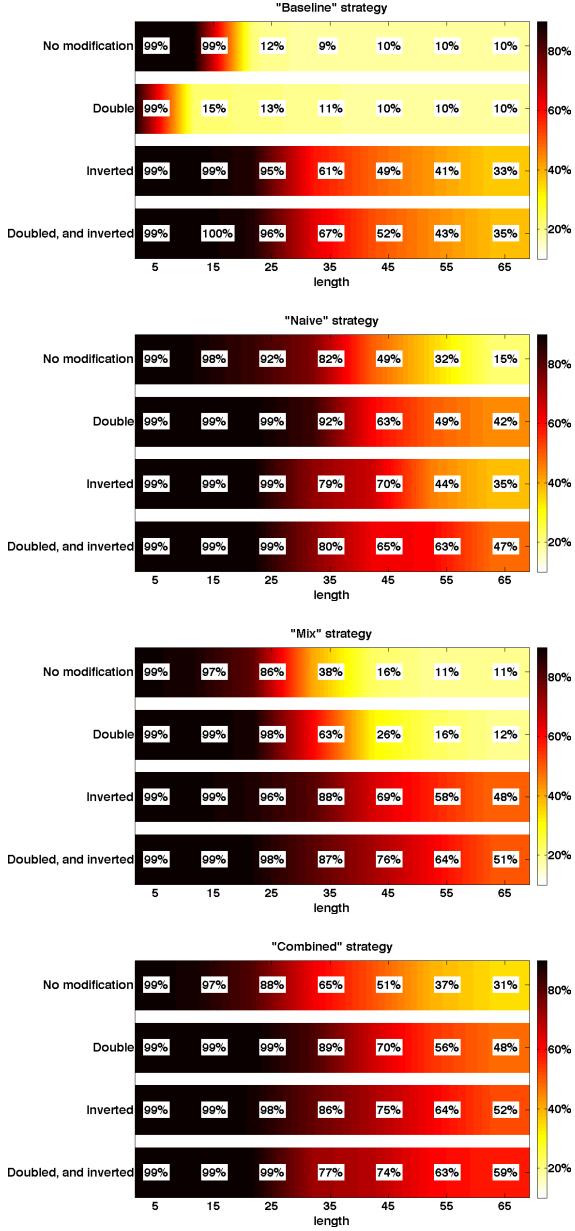


Figure 7. Prediction accuracy on the memorization task for the four curriculum strategies. The input length ranges from 5 to 65 digits. Every strategy is evaluated with the following 4 input modification schemes: no modification; input inversion; input doubling; and input doubling and inversion. Training time was limited to 20 epochs.

of our problems (Sections 6.2 and 6.1) show that the *combined* strategy is better than all other curriculum strategies, including both *naive* curriculum learning, and training on the target distribution. We have a plausible explanation for why this is the case.

It seems natural to train models with examples of increasing difficulty. This way the models have a chance to learn the proper intermediate concepts and input-output mapping, and then utilize them for the more difficult problem instances. Learning the target task might be just too difficult with SGD from a random parameter initialization. This explanation has been proposed in previous work on curriculum learning (Bengio et al., 2009). However, based on empirical results, the *naive* strategy of curriculum learning can sometimes be worse than learning using just with the target distribution.

In our tasks, the neural network has to perform a lot of memorization. The easier examples usually require less memorization than the hard examples. For instance, in order to add two 5-digit numbers, one has to remember at least 5 digits before producing any output. The best way to accurately memorize 5 numbers could be to spread them over the entire hidden state / memory cell (i.e., use a *distributed representation*). Indeed, the network has no incentive to utilize only a fraction of its state. It is always best to make use of its entire memory capacity. This implies that the harder examples would require a restructuring of its memory patterns. It would need to contract its representations of 5 digit numbers in order to free space for the 6-th number. This process of memory pattern restructuring might be difficult to achieve, so it could be the reason for the relatively poor performance of the *naive* curriculum learning strategy (relative to *baseline*).

The *combined* strategy avoids the abrupt problem of restructuring memory patterns. *combined* is a mixture of *naive* curriculum learning strategy and of balanced mixture of examples of all difficulties. The examples produced by the *naive* curriculum strategy help to learn the intermediate input-output mapping, which is useful for solving the target task. The extra samples of all difficulties prevent the network from utilizing all the memory on the easy examples, thus eliminating the need to restructure the memory patterns.

## 8. Critique

Perfect prediction of program output requires an exact understanding of all operands and concepts. However, imperfect prediction might be achieved in a multitude of ways, and could heavily rely on memorization, without a genuine understanding of the underlying concepts. For instance, perfect addition is relatively intricate, as the LSTM needs

to know the order of numbers and to correctly compute the carry.

There are many alternatives to the addition algorithm if perfect output is not required. For instance, one can perform element-wise addition, and as long as there is carry then the output would be perfectly correct. Another alternative, which requires more memory, but is also more simpler, is to memorize all results of addition for 2 digit numbers. Then multi-digit addition can be broken down to multiple 2-digits additions element-wise. Once again, such an algorithm would have a reasonably high prediction accuracy, although it would be far from correct.

We do not know how heavily our model relies on memorization and how far the learnt algorithm is from the actual, correct algorithm. This could be tested by creating a big discrepancy between the training and test data, but in this work, the training and the test distributions are the same. We plan to examine how well our models would generalize on very different new examples in future work.

## 9. Discussion

We have shown that it is possible to learn to evaluate programs with limited prior knowledge. This work demonstrate the power and expressiveness of LSTMs. We also showed that proper curriculum learning is crucial for getting good results on very difficult tasks that cannot be optimized with conventional SGD.

We also found that the general method of doubling the input reliably improves the performance of LSTMs.

Our results are encouraging but they leave many questions open, such as learning to execute generic programs (e.g., ones that run in more than  $O(n)$  time). This cannot be achieved with conventional RNNs or LSTMs due to their runtime restrictions. We also do not know the optimal curriculum learning strategy. To understand that, we may need to identify those training samples that are most beneficial to the model.

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